An Algebraic Method of Balancing Equations
(From Chemistry by Tom Thomson)

In balancing equations we require that the same number of atoms of each element appear on both sides of the equation. The problem is more mathematical than chemical. As you might expect, these equations can be solved by general mathematical methods.

Let us consider the following equation, writing it with the undetermined coefficients a, b, x, and y as shown by:

$$a\ C_2H_6 + b\ O_2 \rightarrow x\ CO_2 + y\ H_2O$$

In order to determine the values of the four unknown coefficients, we must have four equations. These are supplied by the simultaneous balancing of each element. For example, we might start with the element carbon. One equation is obtained by equating the number of carbon atoms on the left side of the equation with those on the right. According to its formula each ethane molecule contains two carbon atoms. Therefore, there are $2a$ carbon atoms in a ethane molecules. On the right side, each carbon dioxide molecule contains only one carbon atoms. Therefore there are x carbon atoms on the right. Since these numbers must be equal at the end of our calculations we write the equation

$$2a = x \quad \text{(Equation 1)}$$

In similar manner, we can write an equation by algebraically balancing the number of hydrogen atoms:

$$6a = 2y \quad \text{(Equation 2)}$$

And doing the same for oxygen, we obtain

$$2b = 2x + y \quad \text{(Equation 3)}$$

This gives us three equations. For a simultaneous algebraic solution we need one more. Since all of the numbers representing atoms are relative, we can obtain our fourth equation by letting any *one* of the unknowns *a, b, x*, or *y* equal anything we wish. If we make a lucky choice, the answers will all come out whole numbers. If we choose too small a number, some fractions will appear which can be rationalized by multiplying all the answers by the appropriate whole number. If our number is too large then we get larger whole numbers than are necessary. We can reduce these to simplest numbers by dividing the equation through by an appropriate number. We will continue our example to illustrate this point.

Let us arbitrarily let $a = 1$ to set up our fourth equation. Then, from Equation 1, $x = 2$. Likewise, from Equation 2, $y = 3$. Placing these values for x and y into Equation 3, we get:

$$2b = 2(2) + 3 = 7$$
$$b = 3.5$$

To avoid fractions, we then multiply through by 2, getting a new set of values for the four coefficients: a = 2, b = 7, x = 4, and y = 6. If you plug in these numbers into the chemical equation you will find that it is correctly balanced.